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Nonlinear three wave interaction in a beam plasma system

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Nonlinear wave-wave interaction is studied in a beam plasma system. Contrary to the usual case of a pure ordinary and an extraordinary waves two mixed longitudinal and transverse modes are excited within the system. A new type of instability due to collisions is found to occur which may be observed for large velocity of the beam.

1. INTRODUCTION

There is a growing interest (Carr *et al* 1972, Chang *et al* 1971) for the generation and amplification of electrostatic waves by means of nonlinear interactions between unstable modes in a beam plasma system. Radiation of transverse waves has also been studied in a beam plasma system by Kovrizhnykh *et al* (1964) and Gailitis *et al* (1964). Dipole radiation is emitted by an electron beam passing through a plasma due to the oscillations of the beam electrons. The dipole radiation may be unstable as a result of nonlinear wave-wave interactions with the plasma waves. This instability of transverse waves arising from the nonlinear interactions of the waves is of interest from the point of view of its application to astrophysical problem (Ginzberg *et al* 1959) and also for the possibility of generation of microwaves (Fainberg *et al* 1957) using a beam plasma system.

This investigation deals with a simple formulation of the beam-plasma problem. The plasma is hot and the beam is also hot but its intensity is small. The beam velocity and the magnetic field are assumed to be parallel to each other. It is assumed that the beam and the stationary plasma are of uniform density.

The relationships have been formulated for determining the frequency and the rate of growth of waves which may be excited by an electron beam. It is observed that in describing a beam-plasma instability one may differentiate between the effects which are directly dependent on the character of the stationary plasma and the effects dependent on the characteristics of the beam e.g., beam velocity. A new type of instability due to collision is found to occur at large velocity of the beam.

2. Basic Equations

The equations are established with the assumptions that the plasma is constituted by electrons neutralised by heavy ions. The ions assure average charge neutrality but do not participate in conduction.

The force equation (Weyl & Goldman 1969) for each kind of electron is

$$\frac{\partial \boldsymbol{v}_\alpha}{\partial t}+\boldsymbol{v}_\alpha.\nabla \boldsymbol{v}_\alpha = -\frac{e}{m_\alpha}\left[\boldsymbol{E}+\frac{1}{c}\,\mathrm{v}_\alpha \times \boldsymbol{B}\right]-\nu \boldsymbol{v}_\alpha-\frac{\chi}{m}\,\frac{\nabla(T_\alpha \rho_\alpha)}{\rho_\alpha}, \qquad \dots \quad (1)$$

where ν is the collision frequency and $\alpha = P$ and $\alpha = b$ refer to the plasma and the beam respectively and χ is the Boltzmann's constant.

The continuity equation is

$$\frac{\partial \rho_\alpha}{\partial t}+\nabla\cdot \boldsymbol{J}_\alpha = 0. \qquad \dots \quad (2)$$

The Maxwell equations are

$$\begin{aligned} \nabla\cdot \boldsymbol{B} &= 0, \\ \nabla\cdot \boldsymbol{E} &= -4\pi e(n_p+n_b), \\ \nabla\times \boldsymbol{B} &= \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}+\frac{4\pi}{c}\,\boldsymbol{J}, \\ \nabla\times \boldsymbol{E} &= -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \end{aligned} \qquad \dots \quad (3)$$

with

$$\boldsymbol{J}_\alpha = \sum_\alpha \rho_\alpha \boldsymbol{v}_\alpha.$$

The equation of state

$$\frac{1}{p}\frac{dp}{dt} = \frac{\gamma}{\rho}\frac{d\rho}{dt}. \qquad \dots \quad (4)$$

The scalar pressure p satisfies the condition $p = n\chi T$ with γ the ratio of specific heats at constant pressure and constant volume. In perturbation expansion the quantiries $\boldsymbol{v}_\alpha$, ρ_α, ..., are expanded in series as

$$\begin{aligned} \boldsymbol{v}_\alpha &= \boldsymbol{v}_\alpha^{(0)}+\boldsymbol{v}_\alpha^{(1)}+\boldsymbol{v}_\alpha^{(2)}+\dots, \\ \rho_\alpha &= \rho_\alpha^{(0)}+\rho_\alpha^{(1)}+\rho_\alpha^{(2)}+\dots, \end{aligned}$$

etc.

The zero order quantities are

$$\boldsymbol{v}_p^{(0)} = 0, \quad \boldsymbol{v}_b^{(0)} = U_0\hat{\boldsymbol{Z}}, \quad \boldsymbol{E}^{(0)} = 0, \quad \boldsymbol{B}^{(0)} = B_0\hat{\boldsymbol{Z}}. \qquad \dots \quad (5)$$

3. First Order Solution

Assuming a space and time dependence of the first order quantities to be of the form $\exp(iK.r-i\omega t)$ we get

$$\rho_\alpha^{(1)} = \frac{\rho_{\alpha 0}{}^0 \boldsymbol{k}\cdot\boldsymbol{v}_\alpha{}^{(1)}}{\omega} \qquad \dots (6)$$

and

$$-i\bar{\omega}\boldsymbol{v}_\alpha{}^{(1)}+\Omega(\hat{x}\hat{y}-\hat{y}\hat{z}).\boldsymbol{v}_\alpha{}^{(1)}+\frac{i\gamma\chi T_{\alpha 0}}{m\omega}\boldsymbol{K}\boldsymbol{K}\cdot\boldsymbol{v}_\alpha{}^{(1)} = -\frac{e}{m}\left[\boldsymbol{E}+\frac{\mathrm{U}_0\times \boldsymbol{B}^{(1)}}{c}\right], \qquad \dots (7)$$

where

$$\bar{\omega} = \omega+i\nu \text{ and } \Omega \text{ (the cyclotron frequency)} = \frac{eB_0}{mc}.$$

Eq. (7) can be solved to obtain

$$\boldsymbol{v}_\alpha{}^{(1)} = \bar{\mu}_\alpha\cdot\boldsymbol{E}^{(1)}, \qquad \dots (8)$$

when

$$\bar{\mu}_\alpha = -\frac{e}{m}\,\mu_\alpha\left[I+\frac{\boldsymbol{K}\boldsymbol{U}_0}{\omega}\right], \qquad \dots (9)$$

$$\mu_\alpha = \frac{i}{\bar{\omega}\left(\Omega^2-\bar{\omega}^2+\beta_\alpha\dfrac{\bar{\omega}}{\omega}\right)}\left[\left(-\bar{\omega}^2+\beta_\alpha\frac{\bar{\omega}}{\omega}\right)\hat{x}\hat{x}+i\bar{\omega}\Omega(\hat{x}\hat{y}-\hat{y}\hat{x})\right.$$

$$\left.\bar{\omega}^2\hat{y}\hat{y}+\left(\Omega^2-\bar{\omega}^2+\beta_\alpha\frac{\bar{\omega}}{\omega}\right)\hat{z}\hat{z}\right], \qquad \dots (10)$$

and

$$I = \text{unit tensor}, \qquad \beta_\alpha = \frac{\chi T_\alpha \gamma K^2.}{m}$$

Fourier transforms of first order fields of the Maxwell equations lead to

$$D.\boldsymbol{E}^{(1)} = 0, \qquad \dots (11)$$

where

$$D(K,\omega) = \left(K^2-\frac{\omega^2}{c^2}\right)I-\boldsymbol{K}\boldsymbol{K}+\frac{i\omega}{c^2}\left[\sum_\alpha \omega_\alpha{}^2\mu_\alpha\left(I+\frac{\boldsymbol{K}\boldsymbol{U}_0}{\omega}\right)+\frac{\omega_b{}^2\boldsymbol{U}_0\boldsymbol{K}\cdot\mu_b}{\omega}\right], \qquad \dots (12)$$

with

$$\omega_\alpha{}^2 = \frac{4\pi e\rho_\alpha{}^{(0)}}{m}, \text{ the plasma frequency.}$$

In case of propagation perpendicular to $\boldsymbol{B}_0$ the dispersion relation is obtained as

$$\text{Det } D(\boldsymbol{K},\omega) = 0. \qquad \dots (13)$$

Eq. (13) has two independent solutions corresponding to the ordinary and extraordinary modes. The solutions are given by

$$c^2K^2 = \omega^2-\omega_p{}^2-\omega_b{}^2+\left(\frac{U_0}{c}\right)^2 \frac{K^2\omega_b{}^4}{(\Omega^2-\omega^2)(\omega_p{}^2+\omega_b{}^2)}+\frac{i\nu}{\omega}(\omega_p{}^2+\omega_b{}^2), \quad \dots \quad (14)$$

$$c^2K^2 = \left[\omega^2-\Sigma\omega_\alpha{}^2+\frac{\Omega^2\Sigma\omega_\alpha{}^2}{\Omega^2+\sum\limits_\alpha \omega_\alpha{}^2-\omega^2}\right]\left\{1+\frac{\Sigma b_\alpha\omega_\alpha{}^2(\Omega^2+\omega^2)}{c^2(\Omega^2-\omega^2)}\right.$$

$$-\frac{2\Omega^2\Sigma\omega_\alpha{}^2b_\alpha(\omega_p{}^2+\omega_b{}^2)}{c^2(\Omega^2-\omega^2)(\Omega^2+\Sigma\omega_\alpha{}^2-\omega^2)}-\frac{\Sigma b_\alpha\Sigma\omega_\alpha{}^2}{(\Omega^2-\omega^2)(\Omega^2+\Sigma\omega_\alpha{}^2-\omega^2)}$$

$$\times\left[\omega^2-\sum_\alpha\omega_\alpha{}^2+\frac{\Omega^2\sum\limits_\alpha\omega_\alpha{}^2}{\Omega^2+\sum\limits_\alpha\omega_\alpha{}^2-\omega^2}\right\}^{-1} \text{with } b_\alpha = \chi T_\alpha\gamma/m_\alpha \quad \dots \quad (15)$$

for the ordinary and the extraordinary waves respectively.

The electric fields corresponding to these modes can be written as

$$\boldsymbol{E}_0 = \hat{a}_0A_0 \exp i(\boldsymbol{k}_0\boldsymbol{r}-\omega_0t)$$
$$\boldsymbol{E}_e = \hat{a}_eA_e \exp i(K_e.r-\omega_et), \quad \dots \quad (16)$$

with K_0 and K_e given by eqs. (14) and (15) respectively, and

$$\hat{a}_0 = \frac{il_0\hat{x}+\hat{z}}{\sqrt{1+l_0{}^2}} \quad \dots \quad (17)$$

$$\hat{a}_e = \frac{il_e\hat{x}+\hat{y}+n_e\hat{z}}{\sqrt{1+l_e{}^2+n_e{}^2}}, \quad \dots \quad (18)$$

with

$$l_0 = -\frac{\omega_b{}^2cK}{\Omega(\omega^2{}_p+\omega_b{}^2)}\left(\frac{U_0}{c}\right)$$

$$l_e = \frac{\omega[(\Omega^2-\omega^2+\omega_p{}^2+\omega_b{}^2)-\sum\frac{\gamma\omega_\alpha{}^2K^2\chi T_\alpha}{m(\Omega^2-\omega^2)}}{\Omega[(\omega_p{}^2+\omega_b{}^2)-\sum\frac{\gamma\omega_\alpha{}^2K^2\chi T_\alpha}{m(\Omega^2-\omega^2)}} \quad \dots \quad (19)$$

$$n_e = \frac{\omega_b{}^2(\Omega^2-\omega^2+\omega_p{}^2+\omega_b{}^2)cK\omega}{\Omega^2(\omega_p{}^2+\omega_b{}^2)^2}\left(\frac{U_0}{c}\right).$$

It may be noted that modes are not pure ordinary or extraordinary as found in cold collisionless plasma, The modes are mixed longitudinal and transverse waves. There are four complex solutions or branches $K = K_r+iK_i$ for each real ω corresponding to the two types of electronic waves—plasma waves and electromagnetic waves. A plot of the dispersion relation has been given in figure 1. The resonance conditions are satisfied in the region denoted by open circle.

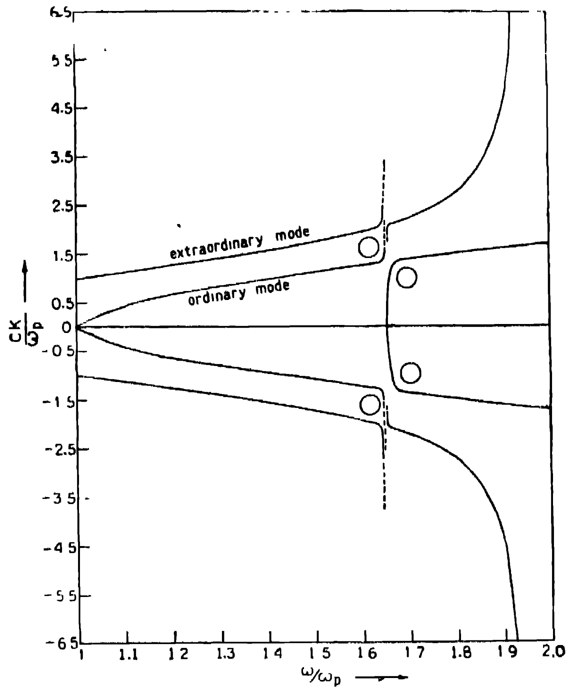

Fig. 1. Dispersion diagram for beam-plasma instabilities in a magnetic field showing K solution for real ω. Resonance conditions are satisfied in the vicinity of four open circles where inequality, $|K_1+K_2|<|K_1|+|K_2|$ is satisfied.

4. Second Order Solution

In a similar way one can develop the second order equations. We solve the equation for

$$\frac{\partial}{\partial t}\boldsymbol{v}_\alpha^{(2)}+\boldsymbol{v}_\alpha^{(1)}\cdot\nabla\boldsymbol{v}_\alpha^{(1)}=-\frac{e}{m}\left[\boldsymbol{E}^{(2)}+\frac{1}{c}\boldsymbol{v}_\alpha^{(2)}\times\boldsymbol{B}_\alpha^{(0)}+\frac{1}{c}\boldsymbol{v}_\alpha^{(1)}\times\boldsymbol{B}^{(1)}+\right.$$

$$\left.+\frac{1}{c}\boldsymbol{U}_0\times\boldsymbol{B}^{(2)}\right]-\nu\boldsymbol{v}_\alpha^{(2)}-\frac{\chi}{m}\left(\frac{\nabla T_\alpha\rho_\alpha}{\rho_\alpha}\right)^{(2)} \quad \dots \quad (20)$$

where

$$\left(\frac{\chi}{m}\frac{\nabla(T_\alpha\rho_\alpha)}{\rho_\alpha}\right)^{(2)}=\frac{-i\chi T_{\alpha_3}}{m}\left[\gamma\nabla\cdot\nabla\boldsymbol{v}_\alpha^{(2)}+\frac{\gamma\nabla\nabla\cdot(\boldsymbol{v}_\alpha^{(1)}\rho_\alpha^{(1)})}{\omega\rho_\alpha^{(0)}}\right.$$

$$\left.-\frac{i\gamma\rho_1\nabla\rho_1}{\rho_0^2}+\frac{i(\gamma-1)}{2}\frac{\gamma\nabla\rho_1^2}{\rho_0^2}\right],$$

to obtain

$$\boldsymbol{v}_\alpha^{(2)}=-\frac{e}{m}\mu_\alpha\cdot\left(I+\frac{\boldsymbol{K}\boldsymbol{U}_0}{\omega}\right)\cdot\boldsymbol{E}^{(2)}-\frac{e}{m}\mu_\alpha\cdot\boldsymbol{Q}, \quad \dots \quad (21)$$

where

$$Q = \frac{1}{c}(\boldsymbol{v}_\alpha^{(1)} \times \boldsymbol{B}^{(1)}) + \frac{m}{e}(\boldsymbol{v}_\alpha^{(1)} \cdot \nabla \boldsymbol{v}_\alpha^{(1)}) + \frac{i\gamma\chi T_\alpha}{m}\left[-\frac{\nabla\nabla\cdot(\boldsymbol{v}_\alpha^{(1)}\rho_\alpha^{(1)})}{\omega\rho_{\alpha_0}}\right.$$

$$\left. + \frac{i\rho_1\nabla\rho_1}{\rho_{\alpha,0}^2} - i\frac{(\gamma-1)}{2}\frac{\nabla\rho_1^2}{\rho_0^2}\right]. \qquad \ldots \quad (22)$$

Eliminating $\boldsymbol{B}^{(2)}$ from the second order equations one may obtain

$$\nabla \times (\nabla \times \boldsymbol{E}^{(2)}) + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}^{(2)}}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial}{\partial t}\boldsymbol{J} = 0. \qquad \ldots \quad (23)$$

where

$$\boldsymbol{J} = \Sigma(\rho_\alpha^{(0)}\boldsymbol{v}_\alpha^{(2)} + \rho_\alpha^{(1)}\boldsymbol{v}_\alpha^{(1)}) + \rho_b^{(2)}\boldsymbol{U}_0. \qquad \ldots \quad (24)$$

Assuming $E^{(2)}$ in the form

$$\boldsymbol{E}^{(2)} = \hat{a}A(y, t)\exp i(Ky - \omega t), \qquad \ldots \quad (25)$$

where $\hat{a}$ is a unit vector and $A(y, t)$ is a slowly varying function of y, and t, eq. (23) can be written in the form

$$\left[\left(K^2 - \frac{\omega^2}{c^2}\right) I - K^2\hat{y}\hat{y} + \frac{i\omega}{c^2}\left\{\sum_\alpha \omega_\alpha^2\mu_\alpha\cdot\left(I + \frac{\boldsymbol{K}\boldsymbol{U}_0}{\omega}\right) + \omega_b^2\boldsymbol{U}_0\boldsymbol{K}\cdot\mu_b\right\}\right].$$

$$\cdot\ E^{(2)} - \left[\frac{4\pi}{c^2}\Sigma(\rho_\alpha\bar{\mu}_\alpha) + \frac{2i\omega}{c^2}I\right]\cdot\hat{a}\frac{\partial A}{\partial t}\exp(iKy - i\omega t) - 2iK[I - \hat{y}\hat{y}].$$

$$\times\hat{a}\frac{\partial A}{\partial y}\cdot\exp(iKy - i\omega t) = \frac{4\pi i\omega}{C^2}\tilde{\boldsymbol{J}}. \qquad \ldots \quad (26)$$

where

$$4\pi\hat{J} = \Sigma\,\omega_\alpha^2\left[\mu_\alpha\cdot\boldsymbol{Q} + \frac{m}{e}\left(\frac{\boldsymbol{K}\cdot\boldsymbol{v}_\alpha^{(1)}}{\omega}\right)\mathrm{v}_\alpha^{(1)}\right] + \frac{4\pi ie\boldsymbol{K}}{c^2}\cdot(\rho_b^{(1)}\boldsymbol{v}_b^{(1)})\boldsymbol{U}_0. \qquad \ldots \quad (27)$$

Since $D(\boldsymbol{K}, \omega) = 0$ dropping the first term of eq. (26) and taking the dot product with a^* one can get

$$\frac{1}{W}\frac{\partial A}{\partial t} + \frac{\partial A}{\partial y} = -\frac{4\pi\omega}{C^2}\frac{a^*\cdot\tilde{J}}{2K\left[1 - \left|\frac{K\hat{a}}{K}\right|^2\right]}, \qquad \ldots \quad (28)$$

where $1/W$ is written ifor

$$\frac{1}{W} = \frac{\omega}{c^2K}\frac{a^*\cdot\left[I - \frac{2\pi i}{\omega}\Sigma\,\rho_\alpha\mu_\alpha\cdot\right]\hat{a}}{\left[1 - \left|\frac{K\cdot a}{K}\right|^2\right]}. \qquad \ldots \quad (29)$$

We apply eq. (28) for each of the three interacting waves i.e.,

$$\left(\frac{\partial}{\partial y}+\frac{1}{W_1}\frac{\partial}{\partial t}\right) A_1 = V(\omega_1 | \omega_2 | \omega_3) A_2 A_3, \qquad \dots (30)$$

where we have used $\boldsymbol{K}_1 = \boldsymbol{K}_2+\boldsymbol{K}_3$, $\omega_1 = \omega_2+\omega_3$

and

$$V = (\omega_1 | \omega_2 | \omega_3) = -\frac{\omega_1 4\pi \hat{a}_1{}^* . \tilde{J}(2,3)}{2c^2\left(1-\left|\frac{K_1 . \hat{a}_1}{K_1}\right|^2\right)} \qquad \dots (31)$$

and

$$4\pi\hat{a}_1{}^*.\bar{J}_{2,3} = \sum_\alpha \omega_\alpha{}^2 \left\{ \frac{1}{\omega_3} (\hat{a}_1{}^*.\mu_{\alpha 1}.\boldsymbol{K}_3)(\hat{a}_3.\mu_{\alpha 2}.\hat{a}_2) - \frac{1}{\omega_3}(\hat{a}_1{}^*.\mu_{\alpha 1}.\hat{a}_3)(\boldsymbol{K}_3.\mu_{\alpha 2}.\hat{a}_2) \right.$$

$$+ i\frac{m}{e}(\hat{a}_1{}^*.\mu_{\alpha 1})(\mu_{\alpha 3}.\hat{a}_3)(\boldsymbol{K}_3.\mu_{\alpha 2}.\hat{a}_2)$$

$$+\frac{i\chi\gamma T_\alpha{}^{(0)}}{m\omega_3}(\boldsymbol{K}_3.\mu_{\alpha 3}.\hat{a}_3)\left[\frac{1}{\omega_1}(\hat{a}_1{}^*.\mu_{\alpha 1}.\boldsymbol{K}_1)(\boldsymbol{K}_1.\mu_{\alpha 2}.\hat{a}_2)\right.$$

$$\left. +\frac{1}{\omega_2}(\boldsymbol{K}_2.\mu_{\alpha 2}.\hat{a}_2)\left(\left(\frac{\gamma-1}{2}(a_1{}^*.\mu_{\alpha 1}.\boldsymbol{K}_1)\right) - a_1{}^*.\mu_{\alpha 1}.\boldsymbol{K}_3\right)\right]$$

$$\left. +\frac{1}{\omega_2}(\boldsymbol{K}_2.\mu_{\alpha 2}.\hat{a}_2)(\hat{a}_1.\mu_{\alpha 3}.\hat{a}_3)\right\} + \frac{\omega_b{}^2}{\omega_2\omega_3}(\boldsymbol{K}_2.\mu_{b2}.\hat{a}_2)(\boldsymbol{K}_3.\mu_{b3}.\hat{a}_3)(a_1{}^*,\boldsymbol{U}_0)$$

$$+2 \rightleftharpoons 3, \qquad \dots (32)$$

It may be mentioned that using eq. (31) and eq. (32) $V(\omega_1 | \omega_2 | \omega_3)$, can be written in the form

$$V(\omega_1 | \omega_2 | \omega_3) = V^{(0)}(\omega_1 | \omega_2 | \omega_3) + \epsilon V^{(1)}(\omega_1 | \omega_2 | \omega_3) + \dots, \qquad \dots (33)$$

with $$\epsilon = \left(\frac{U_0}{C}\right).$$

In a similar way the other two coupled equations are written as

$$\left(\frac{\partial}{\partial y}+\frac{1}{W_2}\frac{\partial}{\partial t}\right) A_2{}^*(y,t) = V(-\omega_2 | -\omega_1 | \omega_3) A_1 A_3{}^*$$

$$\left(\frac{\partial}{\partial y}+\frac{1}{W_3}\frac{\partial}{\partial t}\right) A_3{}^*(y,t) = V(-\omega_3 | \omega_2 | -\omega_1) A_2 A_1{}^*. \qquad \dots (34)$$

The matrix elements $V(-\omega_2 | -\omega_1 | \omega_3)$ and $V(-\omega_3 | \omega_2 | -\omega_1)$ can be obtained from eq. (32) by interchanging ω_1, ω_2 and ω_1, ω_3. They also can be written in the form of eq. (32). We now give the expression for V obtained from eqs. (32) and (34) for the interaction of two ordinary waves with an extraordinary wave. Let

$$\hat{a}_1 = \hat{a}_3 = \hat{a}_0 \qquad \text{(ordinary waves)}$$

and

$$\hat{a}_2 = \hat{a}_e \qquad \text{(extraordinary wave)} \qquad \ldots \quad (35)$$

where $\hat{a}_0$ and $\hat{a}_e$ have been defined in eqs. (17) and (18).

We obtain

$$V(w_1 \mid w_2 \mid w_3) = \Sigma \frac{\omega_\alpha{}^2 e(\Omega l_{e2} - \bar{\omega}_2) K_2 \omega_1}{mc^2 \omega_2 \omega_3 \left(\Omega^2 - \bar{\omega}_2{}^2 + \beta_\alpha \frac{\omega_2}{\omega_2} \right) \sqrt{1 + l_{e2}{}^2 + n_{e2}{}^2} + 0(\epsilon^2)}$$

$$\approx \frac{e}{m} \frac{(\omega_p{}^2 + \omega_b{}^2) K_2 \omega_1 \Omega}{2c^2 \omega_2 \omega_3 \left\{ \frac{\Omega^2}{\omega_2{}^2} (\omega_p{}^2 + \omega_b{}^2)^2 + \left(\omega_p{}^2 + \omega_b{}^2 + \Omega^2 - \omega_2{}^2)^2 \right) \right\}^{\frac{1}{2}}}$$

$$V(\omega_2 \mid \omega_1 \mid \omega_3) = \frac{\omega_2 \bar{\omega}_3}{K_2} \left(\frac{K_3}{\omega_1 \omega_3} + \frac{K_1}{\omega_3 \omega_1} \right) V(\omega_1 \mid \omega_2 \mid \omega_3) + 0(\epsilon^2)$$

$$V(\omega_3 \mid \omega_2 \mid \omega_1) = \frac{\omega_3 \bar{\omega}_3}{\omega_1 \omega_1} V(\omega_1 \mid \omega_2 \mid \omega_3) + 0(\epsilon^2). \qquad \ldots \quad (36)$$

From eqs. (31) and (34), we have a set of three coupled equations for the amplitudes of three waves A_1, A_2, A_3. In the investigation of convectives instabilities treated earlier by Apel, we take ($\partial/\partial t = 0$) A_3 as a constant amplitude pump wave and seek solutions for A and A_2 in the form $e^{\Gamma_0 y}$ and obtain the growth rate

$$\Gamma_0 = \left[\frac{\omega_1 \omega_2}{4c^4 K_1 K_2 \left(1 - \left| \frac{K_2 \hat{a}_2{}^{\dagger}}{K_2} \right|^2 \right)} \right]^{\frac{1}{2}} \left| V(\omega_1 \mid \omega_2 \mid \omega_3) V(-\omega_2 \mid -\omega_1 \mid \omega_3) \right| \qquad \ldots \quad (37)$$

It may be mentioned that contrary to the case of cold collisionless plasma of Etievant *et al* (1968) nonlinear interactions may take place between two extraordinary waves with an ordinary wave, in a beam plasma system. Let

$$\hat{a}_1 = \hat{a}_{10} \qquad \text{(ordinary wave)}$$

and

$$\hat{a}_2 = \hat{a}_{2e}, \quad \hat{a}_3 = \hat{a}_{3e}, \qquad \text{(extraordinary wave).}$$

In a similar way one can obtain the matrix elements $V(\omega_1 \mid \omega_2 \mid \omega_3)$ $V(-\omega_2 \mid -\omega_1 \mid \omega_3)$ and $V(-\omega_3 \mid \omega_2 \mid \omega_1)$ from eq. (32) for such a system. The leading terms in the expansion of $V(\omega_l, \omega_m, \omega_n)$ for such type of three wave interactions is $\epsilon V^{(1)}(\omega_l, \omega_m, \omega_n)$, $V^{(0)}(\omega_e, \omega_m, \omega_n)$ being zero. In the limiting case of $U_0 = \nu = 0$ they are also zero which is in agreement with the case of cold collisionless plasma of Etievant *et al* (1968).

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